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Abstract

An approximate polarization correction for ionization loss suggested by Sternheimer has been implemented in the galactic cosmic ray transport code (HZETRN) developed at the Langley Research Center. Sample calculations made for the aluminum shield and liquid hydrogen shield show no more than a ±2-percent change in the linear energy transfer (LET) distribution for flux compared with those without polarization correction. This very small change is expected because the effect of polarization correction on the reduction in stopping power of ions with energies above 2 GeV/amu is suppressed by the decrease in galactic cosmic ray ion flux at such high energies.

Introduction

As the space program proceeds into an era of extended manned space operations, the protection of astronauts from galactic cosmic rays (GCR's) becomes an important problem for mission designers. The interaction of high-energy heavy ions originating in deep space with target nuclei results in energy degradation and nuclear fragmentations. These fragmentations produce secondary and subsequent-generation reaction products that alter the elemental and isotopic composition of the transported radiation fields. The transported radiation level and linear energy transfer (LET) distribution inside the spacecraft or any critical organ of interest can be estimated by an engineering-design-oriented, computer-efficient, galactic cosmic ray transport code (HZETRN) developed at the Langley Research Center (ref. 1). Although HZETRN has been widely used, it is continuously being modified for better numerical accuracy (ref. 2) and more accurate physics inputs (ref. 3) so that the weight penalty for shields can be minimized.

The stopping power of target materials for high-energy heavy ions is an important input physics parameter to HZETRN, besides nuclear fragmentation parameters. At energies above a few MeV/amu, electronic stopping power is adequately predicted by Bethe's theory provided appropriate corrections to Bragg's rule, shell corrections, and an effective charge are included (ref. 4). As the input to HZETRN, the high-energy cutoff for the incident GCR spectrum is usually taken to be far beyond the minimum stopping power (≈2 GeV/amu) where Bethe's theory starts to overestimate and worsens as the energy increases. This overestimation can be corrected by considering that the field of the incoming ion projectile is perturbed by the polarized neighboring atoms. Because the incident flux of GCR ions falls off at high energies, Bethe's theory has been used without correction in HZETRN and assumed to be adequate. In this work, an approximate correction for the polarization effect (also known as density effect) is included and the effect of this correction is assessed for an aluminum shield and a liquid hydrogen shield.

Galactic Cosmic Ray Transport Method

When moving through extended matter, heavy ions lose energy through interaction with atomic electrons along their trajectories. On occasion, they interact violently with nuclei of the matter and produce ion fragments moving in the forward direction and low-energy fragments of the struck target nucleus. The transport equations for the short-range target fragments can be solved in closed form in terms of collision density (ref. 5). This step leads to the projectile fragment transport as the remaining problem of interest. In previous work, the projectile ion fragments were treated as if all went straight ahead because of the large kinetic energy possessed by the projectile and the fragmentation mechanism. The straight-ahead approximation is found to be quite accurate for the nearly isotropic cosmic ray fluence (ref. 5).

With the straight-ahead approximation and the target secondary fragments neglected (refs. 5 and 6), the transport equation may be written as

$$\left[\frac{\partial}{\partial x} - \nu_j \frac{\partial}{\partial E}\ S(E) + \sigma_j(E)\right] \phi_j(x, E) = \sum_k \int_E^\infty f_{jk}(E, E')\ \phi_k(x, E')\ dE' \tag{1}$$

where $\phi_j(x, E)$ is the flux of ions of type j with atomic mass A_j and charge Z_j at x moving along the x-axis at energy E in units of MeV/amu, σ_j is the corresponding macroscopic nuclear absorption cross section, $S(E)$ is the stopping power of the protons ($S = S_p$), $f_{jk}(E, E')$ is a differential energy cross section for the production of ion j in collision by ion k, and ν_j is the range scaling parameter that is defined as

$$\nu_j = \frac{Z_j^2}{A_j} \tag{2}$$

The stopping power S is the sum of nuclear stopping power and electronic stopping power S_e, where S_e dominates at high energies. Utilizing the definitions

$$r = \int_0^E \frac{dE'}{S(E')} \tag{3}$$

$$\psi_j(x, r) = S(E)\ \phi_j(x, E) \tag{4}$$

and

$$\bar{f}_{jk}(r, r') = S(E)\ f_{jk}(E, E') \tag{5}$$

allows equation (1) to be written as

$$\left[\frac{\partial}{\partial x} - \nu_j \frac{\partial}{\partial r} + \sigma_j(r)\right] \psi_j(x, r) = \sum_k \int_r^\infty \bar{f}_{jk}(r, r')\ \psi_k(x, r')\ dr' \tag{6}$$

which may be rewritten as (ref. 7)

$$\begin{aligned}\psi_j(x, r) = {}& \exp[-\zeta_j(r, x)]\ \psi_j(0, r + \nu_j x) + \sum_k \int_0^x \int_r^\infty \exp[-\zeta_j(r, z)]\ \bar{f}_{jk}(r + \nu_j z, r') \\ & \times \psi_k(x - z, r')\ dr'\ dz\end{aligned} \tag{7}$$

where the exponential is the integrating factor with

$$\zeta_j(r, t) = \int_0^t \sigma_j(r + \nu_j t')\ dt' \tag{8}$$

Rather simple numerical procedures follow from equation (7). By noting that the first-order nature of equation (1) allows $\psi_j(x, r)$ to be taken as a boundary condition for propagation to larger values of x, one may approximate equation (7) as

$$\begin{aligned}\psi_j(x + h, r) = {}& \exp[-\zeta_j(r, h)]\ \psi_j(x, r + \nu_j h) + \sum_k \int_0^h \int_r^\infty \exp[-\zeta_j(r, z)]\ \bar{f}_{jk}(r + \nu_j z, r') \\ & \times \psi_k(x + h - z, r')\ dz\ dr'\end{aligned} \tag{9}$$

If h is sufficiently small that

$$\sigma_j(r')h \ll 1 \tag{10}$$

then, according to perturbation theory (ref. 7),

$$\psi_k(x+h-z,r') \approx \exp[-\zeta_k(r',h-z)]\ \psi_k[x,r'+\nu_k(h-z)] \tag{11}$$

which may be used to reduce the integral of equation (9), thus resulting in

$$\begin{aligned}\psi_j(x+h,r) \approx{} & \exp[-\zeta_j(r,h)]\ \psi_j(x,r+\nu_j h) \\ & + \sum_k \int_0^h \int_r^\infty \exp[-\zeta_j(r,z) - \zeta_k(r',h-z)]\ \bar{f}_{jk}(r+\nu_j z, r') \\ & \times \psi_k[x,r'+\nu_k(h-z)]\ dr'\ dz \end{aligned} \tag{12}$$

Currently, for $Z_j > 1$ and $k > j$ we assume that

$$\bar{f}_{jk}(r,r') = \sigma_{jk}(r')\ \delta(r-r') \tag{13}$$

By using equation (13), equation (12) now becomes

$$\begin{aligned}\psi_j(x+h,r) \approx{} & \exp[-\zeta_j(r,h)]\ \psi_j(x,r+\nu_j h) \\ & + \sum_k \int_0^h dz\ \exp[-\zeta_j(r,z) - \zeta_k(r',h-z)]\ \sigma_{jk}(r') \\ & \times \psi_k[x,r'+\nu_k(h-z)] \end{aligned} \tag{14}$$

with $r' = r + \nu_j z$. Equation (14) is further approximated as

$$\begin{aligned}\psi_j(x+h,r) \approx{} & \exp[-\zeta_j(r,h)]\ \psi_j(x,r+\nu_j h) \\ & + \sum_k \int_0^h dz\ \exp[-\zeta_j(r,z) - \zeta_k(r,h-z)]\ \sigma_{jk}(r)\ \psi_k[x,r+\nu_j z+\nu_k(h-z)] \\ \approx{} & \exp[-\sigma_j(r)h]\ \psi_j(x,r+\nu_j h) \\ & + \sum_k \sigma_{jk}(r) \left\{ \frac{\exp[-\sigma_j(r)h] - \exp[-\sigma_k(r)h]}{\sigma_k(r) - \sigma_j(r)} \right\} \psi_k(x,r+\nu_j h) \\ & + O[(\nu_k - \nu_j)h] \end{aligned} \tag{15}$$

Equation (15) is the stepping formalism with energy-dependent cross sections for $k > {}^4\mathrm{He}$. The corresponding stepping formalism for nucleons has been discussed in detail in reference 4. These stepping formalisms are then used to march the solution from the surface boundary to the desired shield thickness.

Approximate Polarization Correction

The reduction in the ionization loss of ultrarelativistic charged particles due to the polarization of the medium was first treated by Fermi (ref. 8) who assumed that the dispersive properties can be described by a single type of dispersion oscillator. Halpern and Hall (ref. 9), Wick (ref. 10), and Sternheimer (ref. 11) extended Fermi's equations to the general case of an arbitrary number of dispersion oscillators. In reference 11, Sternheimer evaluated the density-effect (polarization-effect) correction for various elements and compounds and fitted his calculated results by the expressions

$$\delta = 0 \qquad (x < x_o) \tag{16a}$$

$$\delta = \ln \eta^2 + C + a(x_1 - x)^m \qquad (x_o < x < x_1) \tag{16b}$$

$$\delta = \ln \eta^2 + C \qquad (x > x_1) \tag{16c}$$

where δ is the density-effect correction that enters the stopping-power formula, $\eta = p/m_o c$, p is the momentum, m_o is the rest mass of the charged particle, c is the velocity of light, and

$$x = (\log_{10} e) \ln \eta = 0.43429 \ln \eta$$

The quantities a, m, x_o, and x_1 are constants that must be evaluated for each material; C is given by

$$C = -2\ \ln(I/h\nu_p) - 1 \tag{17}$$

where I is the mean ionization potential, $h\nu_p$ (the plasma energy of the material) is given by

$$h\nu_p = h(ne^2/\pi m_e)^{1/2} \tag{18}$$

n is the electron density (in electrons per cubic centimeter), m_e is the rest mass, and e is the charge of an electron. For compounds or mixtures, the effective mean ionization potential can be determined by

$$\ln I = \frac{1}{n} \sum_i n_i \ln I_i \tag{19}$$

where n_i is the electron density for the ith element and I_i is the corresponding atomic ionization potential.

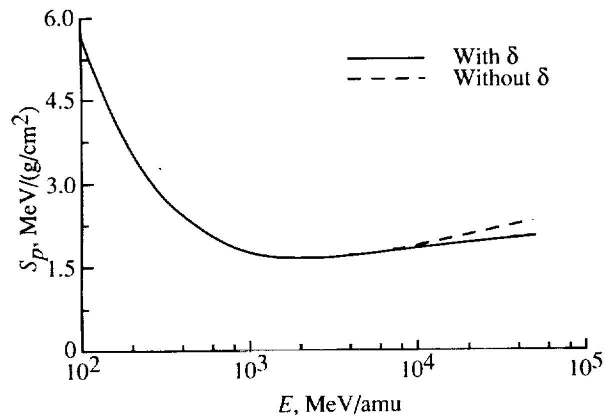
Sternheimer (ref. 11) has suggested that for some practical applications, using only the asymptotic density-effect correction (eq. (16c)) may be adequate for all charged-particle energies (ref. 12). Armstrong and Alsmiller (ref. 12) subsequently compared the differences in stopping power with correct expressions and an asymptotic formula for several elements and compounds. The results indicate an overestimate of, at most, 6 percent when using the asymptotic formula. Therefore, only the asymptotic formula is used for this study. Bethe's high-energy approximation to the energy loss per unit path, with the density-effect correction δ included, is given as

$$S_e = \frac{4\pi N Z_p^2 Z_t e^4}{mv^2} \left\{ \ln \left[\frac{2mv^2}{(1-\beta^2)I} \right] - \beta^2 - \frac{C_s}{Z_t} - \frac{\delta}{2} \right\} \tag{20}$$

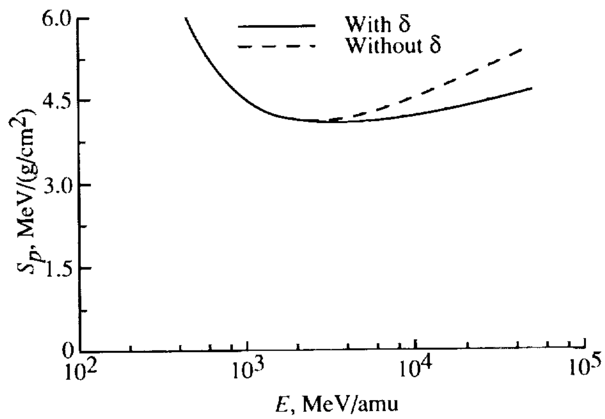
where N is the number of target molecules per unit volume, Z_p is the projectile charge, Z_t is the number of electrons per target molecule, m is the electron mass, v is the projectile velocity, $\beta = v/c$, and C_s is the velocity-dependent shell correction term (ref. 13). Obviously, the use of equation (20) in conjunction with the asymptotic formula (eq. (16c)) does not involve any evaluation of the constants a, m, x_o, and x_1 for the material considered.

Results

Sample calculations are made for an aluminum shield and a liquid hydrogen shield to explore the effect of polarization correction in ionization loss. Figure 1 shows the proton stopping power in these two materials predicted by Bethe's theory with and without Sternheimer's approximate polarization correction. The reduction in stopping power due to the polarization effect occurs at energies beyond the minimum value of the stopping power and increases with the energy. However, the effect of a large reduction in stopping power on the GCR transport calculation may be suppressed by the

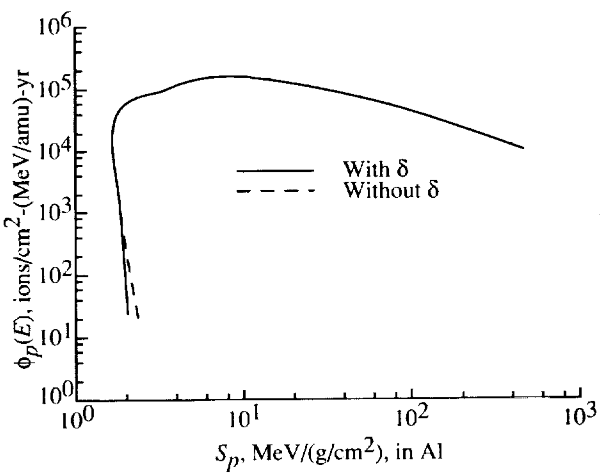


(a) Proton stopping power in aluminum.

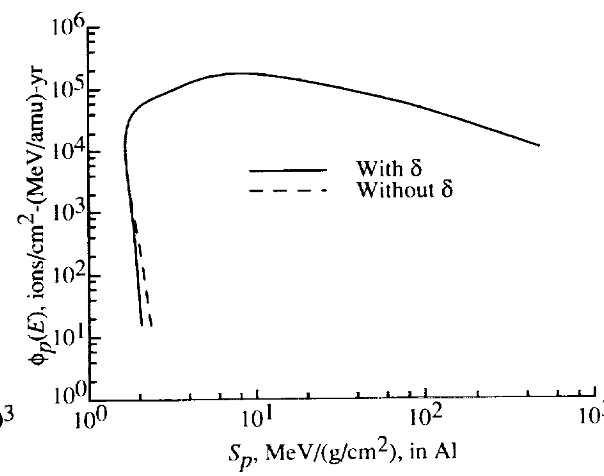


(b) Proton stopping power in liquid hydrogen.

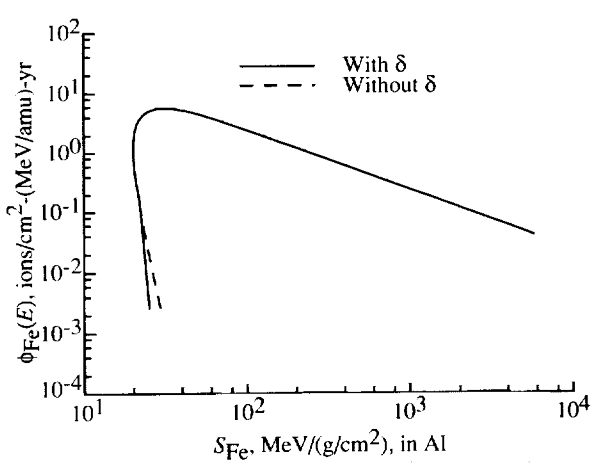
Figure 1. Proton stopping power in aluminum and liquid hydrogen predicted by Bethe's theory with and without Sternheimer's approximate polarization correction.



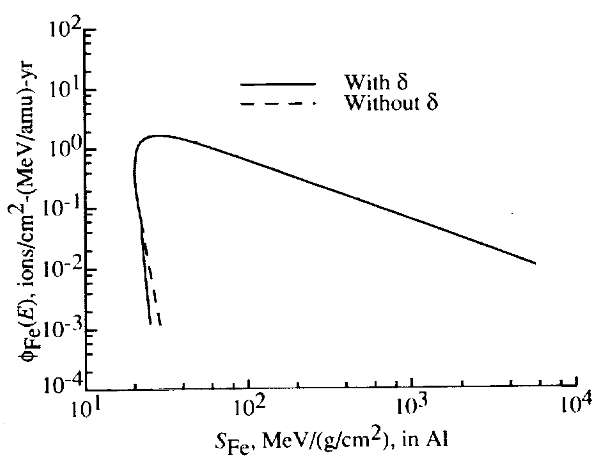
(a) Proton flux ϕ_p through shield thickness of 15 g/cm^2.



(b) Proton flux ϕ_p through shield thickness of 30 g/cm^2.



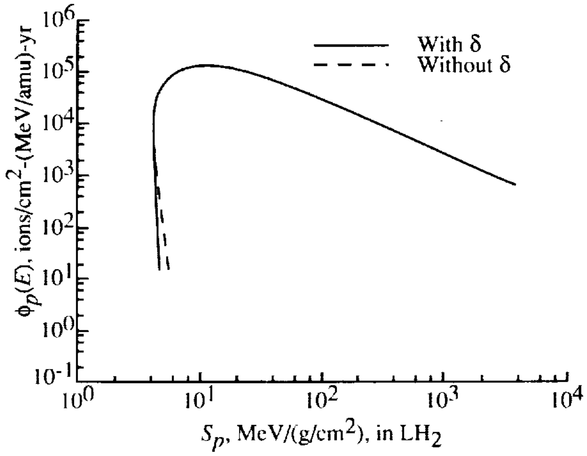
(c) Iron flux ϕ_{Fe} through shield thickness of 15 g/cm^2.



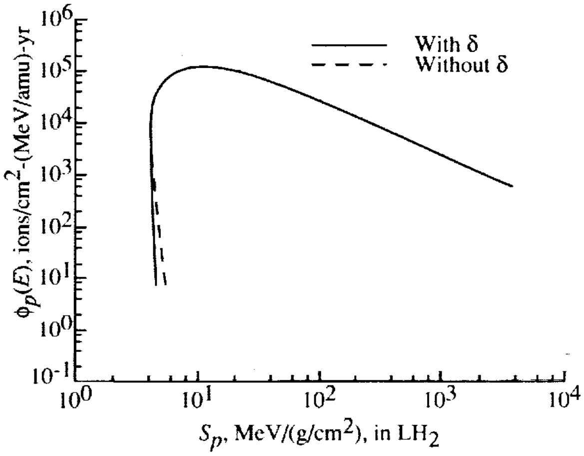
(d) Iron flux ϕ_{Fe} through shield thickness of 30 g/cm^2.

Figure 2. Comparison of transmitted GCR ion flux through aluminum shield at solar minimum with and without Sternheimer's approximate polarization correction. Flux distribution shown as function of ion stopping power in shield.

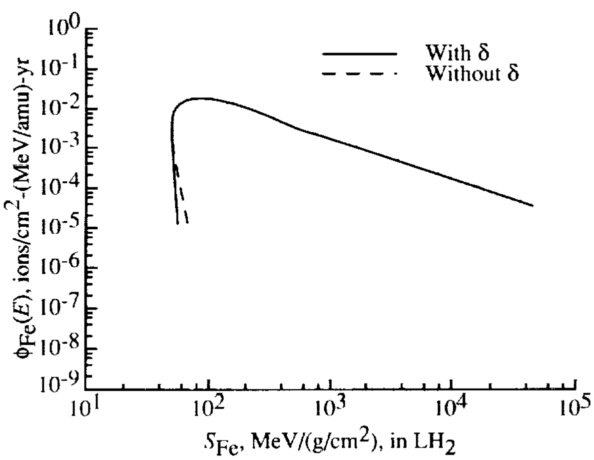
decreasing GCR flux distribution at high energies. Figures 2 5 display the calculated results for GCR transport through the aluminum shield (figs. 2 and 4) and the liquid hydrogen shield (figs. 3 and 5) when using the energy-dependent version of HZETRN. The incident spectrum is based on the CREME (cosmic ray effects on microelectronics) model (ref. 14) at solar minimum. For the transported proton and iron ions, the flux distributions as a function of ion stopping power are shown with and without the polarization correction in an aluminum medium (fig. 2) and liquid hydrogen medium (fig. 3) at a depth of 15 and 30 g/cm^2. The calculated results based on the use of the correction are essentially the same, but, as shown in figure 1, the results begin to diverge beyond the minimum stopping power.



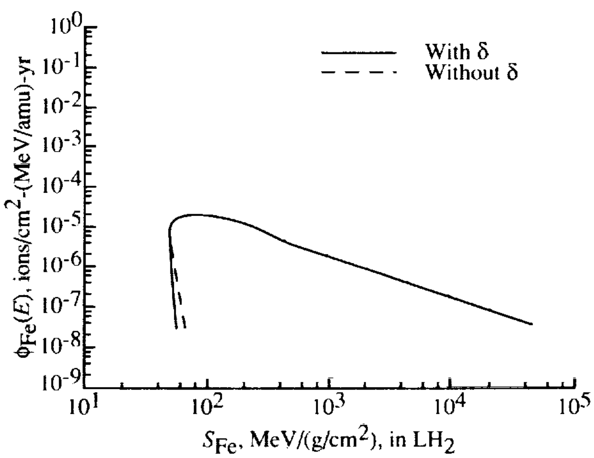
(a) Proton flux ϕ_p through shield thickness of 15 g/cm^2.



(b) Proton flux ϕ_p through shield thickness of 30 g/cm^2.



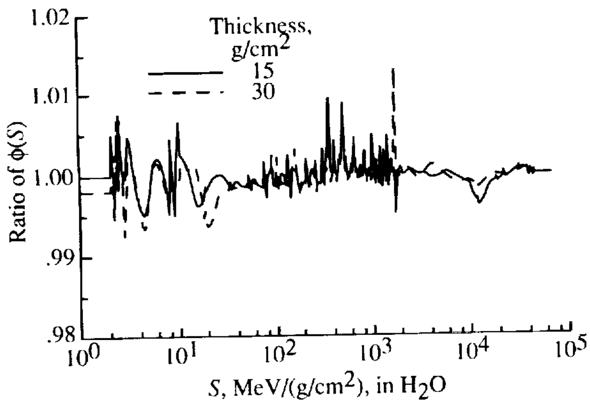
(c) Iron flux ϕ_{Fe} through shield thickness of 15 g/cm^2.



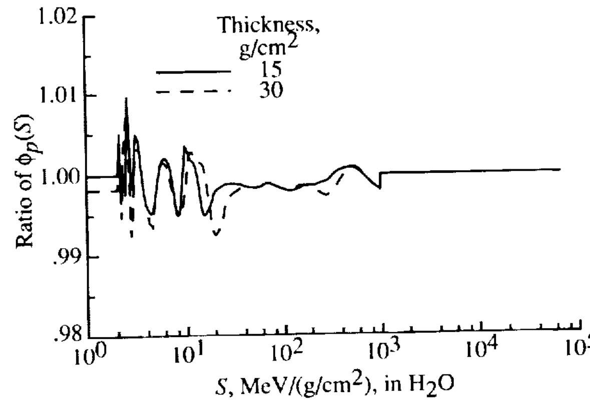
(d) Iron flux ϕ_{Fe} through shield thickness of 30 g/cm^2.

Figure 3. Comparison of transmitted GCR ion flux through liquid hydrogen shield at solar minimum with and without Sternheimer's approximate polarization correction. Flux distribution shown as function of ion stopping power in shield.

Ratios are taken for the LET distributions of the total transported flux through the aluminum shield (fig. 4(a)) and liquid hydrogen shield (figs. 5(a) and 5(c)) of various thicknesses to numerically quantify such a small difference in transport calculation due to the polarization effect. Deviations of, at most, ± 2 percent are noticeable even for thicknesses up to 100 g/cm^2 of liquid hydrogen. Most of the structured peaks come from the differences in the very high energy region (or very low LET region) of each ion species. As more heavy ions are broken up into low LET fragments in penetrating

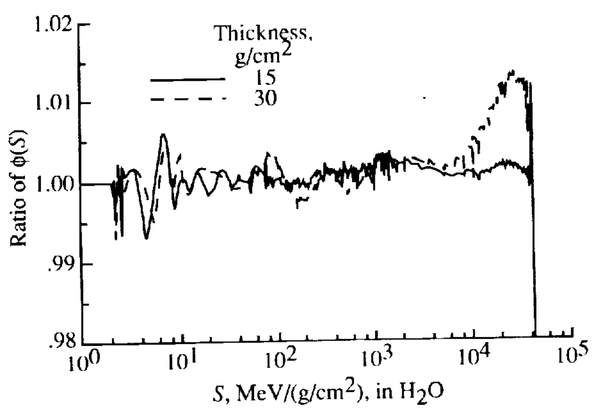


(a) Total ion flux ϕ.

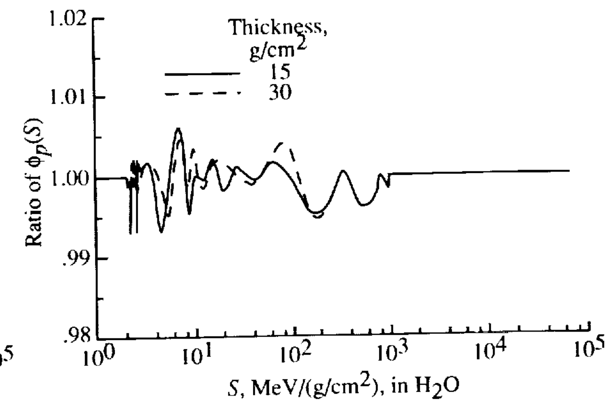


(b) Proton flux ϕ_p.

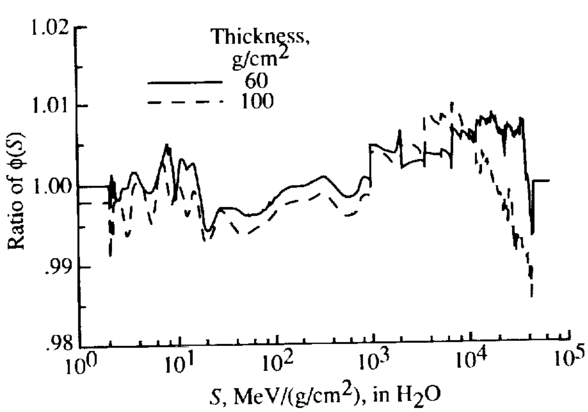
Figure 4. Ratio of differential LET flux calculated with and without Sternheimer's approximate polarization correction. Transmitted GCR flux through aluminum shield thicknesses of 15 and 30 g/cm^2 at solar minimum.



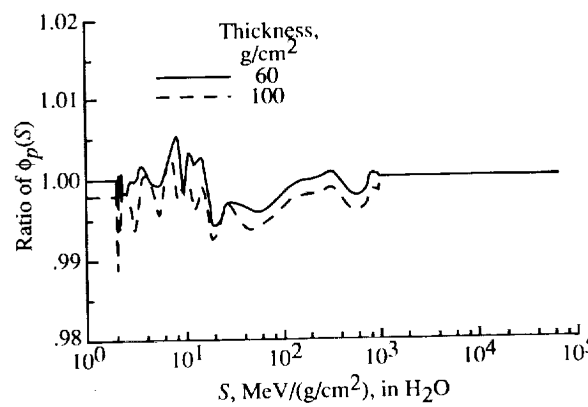
(a) Total ion flux ϕ through shield thicknesses of 15 and 30 g/cm^2.



(b) Proton flux ϕ_p through shield thicknesses of 15 and 30 g/cm^2.



(c) Total ion flux ϕ through shield thicknesses of 60 and 100 g/cm^2.



(d) Proton flux ϕ_p through shield thicknesses of 60 and 100 g/cm^2.

Figure 5. Ratio of differential LET flux calculated with and without Sternheimer's approximate polarization correction. Transmitted GCR flux through liquid hydrogen shield at various thicknesses at solar minimum.

farther into the shield, the low LET ions tend to dominate more strongly. This result is reflected in the similarity of structures for the ratios of total flux and proton flux (fig. 4(b) for aluminum and figs. 5(b) and 5(d) for liquid hydrogen) at the low LET region. The humps seen for the liquid hydrogen shield at high LET for thicknesses near and beyond 30 g/cm^2 are insignificant because most of the heavy ions have disappeared as a result of the large stopping power possessed by liquid hydrogen.

This study concludes that when the full GCR spectrum is considered, the calculated LET spectra behind shields are affected by the polarization correction by no more than 2 percent. However, the correction is expected to have a larger effect on calculated transport results for the cases with geomagnetic cutoff (as in lower Earth orbit) and a much larger effect on laboratory beam experiments with energies greater than 2 GeV/amu.

Concluding Remarks

An approximate polarization correction for ionization loss suggested by Sternheimer has been implemented in the galactic cosmic ray transport code (HZETRN). The effect of a large reduction in the stopping power due to polarization for very high energy ions (beyond 2 GeV/amu) on the galactic cosmic ray (GCR) transport calculation is not more than ±2 percent as has been suspected. Further detailed implementation with a more accurate formula providing further correction in the range from 1 to 10 GeV/amu would be of interest but would provide only a small correction for space radiation application beyond the Earth's magnetosphere.

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